



09/965236 03-26-V.7

def

PTO/SB/17 (02-07)

Approved for use through 02/28/2007. OMB 0651-0032

U.S. Patent and Trademark Office; U.S. DEPARTMENT OF COMMERCE

Under the Paperwork Reduction Act of 1995, no persons are required to respond to a collection of information unless it displays a valid OMB control number.

Effective on 12/08/2004.  
Fees pursuant to the Consolidated Appropriations Act, 2005 (H.R. 4818).**FEE TRANSMITTAL**  
**For FY 2007****Complete if Known**☒ Applicant claims small entity status. See 37 CFR 1.27**TOTAL AMOUNT OF PAYMENT** (\$) 0

Application Number	Patent Number US 7,062,093 B2
Filing Date	Issue Date June 13, 2006
First Named Inventor	Steger
Examiner Name	Jon Chang
Art Unit	2623
Attorney Docket No.	205,286

**METHOD OF PAYMENT** (check all that apply)

- ☐ Check ☐ Credit Card ☐ Money Order ☐ None ☐ Other (please identify): \_\_\_\_\_
- ☒ Deposit Account Deposit Account Number: 01-0035 Deposit Account Name: ABELMAN,FRAYNE&SCHWAB
- For the above-identified deposit account, the Director is hereby authorized to: (check all that apply)
- ☐ Charge fee(s) indicated below ☐ Charge fee(s) indicated below, **except for the filing fee**
- ☒ Charge any additional fee(s) or underpayments of fee(s) under 37 CFR 1.16 and 1.17 ☒ Credit any overpayments

**WARNING:** Information on this form may become public. Credit card information should not be included on this form. Provide credit card information and authorization on PTO-2038.**FEE CALCULATION****1. BASIC FILING, SEARCH, AND EXAMINATION FEES**

Application Type	FILING FEES		SEARCH FEES		EXAMINATION FEES		Fees Paid (\$)
	Fee (\$)	Small Entity Fee (\$)	Fee (\$)	Small Entity Fee (\$)	Fee (\$)	Small Entity Fee (\$)	
Utility	300	150	500	250	200	100	0
Design	200	100	100	50	130	65	0
Plant	200	100	300	150	160	80	0
Reissue	300	150	500	250	600	300	0
Provisional	200	100	0	0	0	0	0

**2. EXCESS CLAIM FEES****Fee Description**

Each claim over 20 (including Reissues) \_\_\_\_\_

Each independent claim over 3 (including Reissues) \_\_\_\_\_

Multiple dependent claims \_\_\_\_\_

Fee (\$)	Small Entity Fee (\$)
	Fee (\$)
50	25
200	100
360	180

<b>Total Claims</b>	<b>Extra Claims</b>	<b>Fee (\$)</b>	<b>Fee Paid (\$)</b>
_____ - 20 or HP = _____	x _____	= _____	0

HP = highest number of total claims paid for, if greater than 20.

<b>Indep. Claims</b>	<b>Extra Claims</b>	<b>Fee (\$)</b>	<b>Fee Paid (\$)</b>
_____ - 3 or HP = _____	x _____	= _____	0

HP = highest number of independent claims paid for, if greater than 3.

**3. APPLICATION SIZE FEE**

If the specification and drawings exceed 100 sheets of paper (excluding electronically filed sequence or computer listings under 37 CFR 1.52(e)), the application size fee due is \$250 (\$125 for small entity) for each additional 50 sheets or fraction thereof. See 35 U.S.C. 41(a)(1)(G) and 37 CFR 1.16(s).

<b>Total Sheets</b>	<b>Extra Sheets</b>	<b>Number of each additional 50 or fraction thereof</b>	<b>Fee (\$)</b>	<b>Fee Paid (\$)</b>
_____ - 100 = _____	/ 50 = _____	(round up to a whole number) x _____	= _____	0

**4. OTHER FEE(S)**

Non-English Specification, \$130 fee (no small entity discount) \_\_\_\_\_

Other (e.g., late filing surcharge): \_\_\_\_\_

Certificate  
of Correction  
MAR 29 2007

<b>SUBMITTED BY</b>		
Signature		Registration No. 36,223 (Attorney/Agent)
Name (Print/Type)	Anthony J. Natoli	Telephone 212-949-9022
		Date 03/22/2007

This collection of information is required by 37 CFR 1.136. The information is required to obtain or retain a benefit by the public which is to file (and by the USPTO to process) an application. Confidentiality is governed by 35 U.S.C. 122 and 37 CFR 1.14. This collection is estimated to take 30 minutes to complete, including gathering, preparing, and submitting the completed application form to the USPTO. Time will vary depending upon the individual case. Any comments on the amount of time you require to complete this form and/or suggestions for reducing this burden, should be sent to the Chief Information Officer, U.S. Patent and Trademark Office, U.S. Department of Commerce, P.O. Box 1450, Alexandria, VA 22313-1450. DO NOT SEND FEES OR COMPLETED FORMS TO THIS ADDRESS. SEND TO: Commissioner for Patents, P.O. Box 1450, Alexandria, VA 22313-1450.

If you need assistance in completing the form, call 1-800-PTO-9199 and select option 2.

APR 2 - 2007



205,286

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

Patentee: Steger Examiner: Jon Chang  
Patent Number: US 7,062,093 B2 Group Art Unit: 2623  
Issue Date: June 13, 2006  
Title: SYSTEM AND METHOD FOR OBJECT RECOGNITION

STATEMENT OF FILING BY EXPRESS MAIL 37 C.F.R. SECTION 1.10

This correspondence is being deposited with the United States Postal Service on March 22, 2007 in an envelope as "Express Mail Post Office to Addressee" Mail Label Number ER 059 678 242 US addressed to the Commissioner for Patents, P.O. Box 1450, Alexandria, VA 22313-1450.

ATTN: Certification of Correction Branch  
Commissioner for Patents  
P.O. Box 1450  
Alexandria, VA 22313-1450

REQUEST FOR EXPEDITED ISSUANCE OF A  
CERTIFICATE OF CORRECTION - ERROR ATTRIBUTABLE TO THE OFFICE

The patentee requests the issuance of a Certificate of Correction in connection with the above-identified patent, as per the attached Forms PTO/SB/44, in which the following revisions are requested:

On the cover page, in the inventor's name,

"Carstan" should read --Carsten--.

Column 2, line 5, "φ" should read -- φ --.

Column 2, line 21, "φ" should read -- φ --.

Column 2, line 24, "φ" should read -- φ --.

Column 10, lines 10-14, the lower portion of the equation

after the second “=” should read:

$$\frac{1}{n} \sum_{i=1}^n \frac{d_i'^x e_{x+p_i'^x, y+p_i'^y}^x + d_i'^y e_{x+p_i'^x, y+p_i'^y}^y}{\sqrt{(d_i'^x)^2 + (d_i'^y)^2} \cdot \sqrt{(e_{x+p_i'^x, y+p_i'^y}^x)^2 + (e_{x+p_i'^x, y+p_i'^y}^y)^2}}$$

Column 11, line 55, “ $\phi_{\min} \leq \phi \leq \phi_{\max}$ ” should read:

$$\omega_{\min} \leq \omega \leq \omega_{\max}$$

Column 13, line 6, “ $\phi$ ” should read --  $\varphi$  --.

Column 14, line 40, “ $\Delta\omega$ ” should read --  $\Delta\varphi$  --.

Column 16, line 25, “ $\phi$ ” should read --  $\varphi$  --.

Column 16, line 44, “ $\phi$ ” should read --  $\varphi$  --.

Column 16, line 46, “ $\phi$ ” should read --  $\varphi$  --.

Column 18, lines 1-4, the equation should read:

$$\frac{1}{m} \sum_{j=1}^m \|Aq_j + t - r_j\|_2 \rightarrow \min$$

Column 21, line 31, “ $\phi_{\min} \leq \phi \leq \phi_{\max}$ ” should read

$$\omega_{\min} \leq \omega \leq \omega_{\max}$$

Column 22, line 48, “ $\phi$ ” should read --  $\varphi$  --.

## REMARKS

The present U.S. patent US 7,062,093 B2 was filed as a non-provisional utility application on November 26, 2001 as U.S. application number 09/965,236. In the U.S. patent US 7,062,093 B2 granted therefrom on June 13, 2006, numerous errors were printed in the issued patent, as set forth herein and in the accompanying Forms PTO/SB/44.

It is respectfully submitted that all of the errors set forth herein and in the accompanying Forms PTO/SB/44 were due solely to an office mistake of the U.S. Patent and Trademark Office. Accordingly, no fees are due.

Regarding the name of the inventor, there is only one inventor, and the correction to the name of the sole inventor is of a typographical nature, which does not affect the inventorship of the issued patent. Support for the correct first name of the inventor to be "Carsten" is clearly shown on the most up-to-date Corrected Filing Receipt dated May 2, 2006, as well as the Notice of Allowance and Fee(s) Due, dated May 9, 2005 (copies enclosed).

As to the text of the issued patent, such errors and discrepancies in the issued patent compared to the text of the application as originally filed by the applicant are clearly disclosed in the records of the U.S. Patent and Trademark Office, as shown in supporting documentation, being copies of the relevant pages of the application as originally filed, attached as an appendix to the present request. The correct text is indicated in red on the attached pages of the application as originally filed, that is:

at page 2, line 20;

at page 3, lines 8 and 10;

at page 13, line 20;

at page 16, line 25;

at page 19, line 18;  
at page 23, line 1;  
at page 26, line 22;  
at page 27, lines 12 and 13;  
at page 30, line 1;  
at page 36, line 8; and  
at page 38, line 4.

Accordingly, expedited issuance of a Certificate of Correction to correct the aforesaid errors and discrepancies is respectfully requested.

Since the aforesaid errors and discrepancies in the issued patent were solely due to an Office Mistake, no fees are required, as per 35 U.S.C. § 254 and 37 C.F.R. § 1.322(b).

In case of any deficiencies in fees by the filing of the present Request for Certificate of Correction, the Commissioner is hereby authorized to charge such deficiencies in fees to Deposit Account Number 01-0035.

Respectfully submitted,



Anthony J. Natoli  
Registration number 36,223  
Attorney for patentee

Date: March 22, 2007

ABELMAN, FRAYNE & SCHWAB  
666 Third Ave., 10th Floor  
New York, NY 10017-5621  
(212) 949-9022

## UNITED STATES PATENT AND TRADEMARK OFFICE CERTIFICATE OF CORRECTION

Page 1 of 3

PATENT NO. : US 7,062,093

APPLICATION NO.: 09/965,236

ISSUE DATE : June 13, 2006

INVENTOR(S) : Carsten Steger

It is certified that an error appears or errors appear in the above-identified patent and that said Letters Patent is hereby corrected as shown below:

On the cover page, in the inventor's name,

"Carstan" should read --Carsten--.

Column 2, line 5, " $\phi$ " should read --  $\varphi$  --.

Column 2, line 21, " $\phi$ " should read --  $\varphi$  --.

Column 2, line 24, " $\phi$ " should read --  $\varphi$  --.

Column 10, lines 10-14, the lower portion of the equation after the second "=" should read:

$$\frac{1}{n} \sum_{i=1}^n \frac{d_i'^x e_{x+p_i'^x, y+p_i'^y}^x + d_i'^y e_{x+p_i'^x, y+p_i'^y}^y}{\sqrt{(d_i'^x)^2 + (d_i'^y)^2} \cdot \sqrt{(e_{x+p_i'^x, y+p_i'^y}^x)^2 + (e_{x+p_i'^x, y+p_i'^y}^y)^2}}$$

MAILING ADDRESS OF SENDER (Please do not use customer number below):

ABELMAN, FRAYNE & SCHWAB  
666 Third Ave., 10th Floor  
New York, NY 10017-5621

This collection of information is required by 37 CFR 1.322, 1.323, and 1.324. The information is required to obtain or retain a benefit by the public which is to file (and by the USPTO to process) an application. Confidentiality is governed by 35 U.S.C. 122 and 37 CFR 1.14. This collection is estimated to take 1.0 hour to complete, including gathering, preparing, and submitting the completed application form to the USPTO. Time will vary depending upon the individual case. Any comments on the amount of time you require to complete this form and/or suggestions for reducing this burden, should be sent to the Chief Information Officer, U.S. Patent and Trademark Office, U.S. Department of Commerce, P.O. Box 1450, Alexandria, VA 22313-1450. DO NOT SEND FEES OR COMPLETED FORMS TO THIS ADDRESS. SEND TO: **Attention Certificate of Corrections Branch, Commissioner for Patents, P.O. Box 1450, Alexandria, VA 22313-1450.**

*If you need assistance in completing the form, call 1-800-PTO-9199 and select option 2.*

**APR 2 - 2007**

UNITED STATES PATENT AND TRADEMARK OFFICE  
CERTIFICATE OF CORRECTION

Page 2 of 3

PATENT NO. : US 7,062,093  
APPLICATION NO.: 09/965,236  
ISSUE DATE : June 13, 2006  
INVENTOR(S) : Carsten Steger

It is certified that an error appears or errors appear in the above-identified patent and that said Letters Patent is hereby corrected as shown below:

Column 11, line 55, " $\phi_{\min} \leq \phi \leq \phi_{\max}$ " should read:

$$\omega_{\min} \leq \omega \leq \omega_{\max}$$

Column 13, line 6, " $\phi$ " should read --  $\varphi$  --.

Column 14, line 40, " $\Delta\omega$ " should read --  $\Delta\varphi$  --.

Column 16, line 25, " $\phi$ " should read --  $\varphi$  --.

Column 16, line 44, " $\phi$ " should read --  $\varphi$  --.

Column 16, line 46, " $\phi$ " should read --  $\varphi$  --.

MAILING ADDRESS OF SENDER (Please do not use customer number below):

ABELMAN, FRAYNE & SCHWAB  
666 Third Ave., 10th Floor  
New York, NY 10017-5621

This collection of information is required by 37 CFR 1.322, 1.323, and 1.324. The information is required to obtain or retain a benefit by the public which is to file (and by the USPTO to process) an application. Confidentiality is governed by 35 U.S.C. 122 and 37 CFR 1.14. This collection is estimated to take 1.0 hour to complete, including gathering, preparing, and submitting the completed application form to the USPTO. Time will vary depending upon the individual case. Any comments on the amount of time you require to complete this form and/or suggestions for reducing this burden, should be sent to the Chief Information Officer, U.S. Patent and Trademark Office, U.S. Department of Commerce, P.O. Box 1450, Alexandria, VA 22313-1450. DO NOT SEND FEES OR COMPLETED FORMS TO THIS ADDRESS. SEND TO: Attention Certificate of Corrections Branch, Commissioner for Patents, P.O. Box 1450, Alexandria, VA 22313-1450.

If you need assistance in completing the form, call 1-800-PTO-9199 and select option 2.

APR 2 - 2007

## UNITED STATES PATENT AND TRADEMARK OFFICE CERTIFICATE OF CORRECTION

Page 3 of 3

PATENT NO. : US 7,062,093

APPLICATION NO.: 09/965,236

ISSUE DATE : June 13, 2006

INVENTOR(S) : Carsten Steger

It is certified that an error appears or errors appear in the above-identified patent and that said Letters Patent is hereby corrected as shown below:

Column 18, lines 1-4, the equation should read:

$$\frac{1}{m} \sum_{j=1}^m \|Aq_j + t - r_j\|_2 \rightarrow \min$$

Column 21, line 31, " $\phi_{\min} \leq \phi \leq \phi_{\max}$ " should read

$$\omega_{\min} \leq \omega \leq \omega_{\max}$$

Column 22, line 48, " $\phi$ " should read --  $\varphi$  --.

MAILING ADDRESS OF SENDER (Please do not use customer number below):

ABELMAN, FRAYNE & SCHWAB  
666 Third Ave., 10th Floor  
New York, NY 10017-5621

This collection of information is required by 37 CFR 1.322, 1.323, and 1.324. The information is required to obtain or retain a benefit by the public which is to file (and by the USPTO to process) an application. Confidentiality is governed by 35 U.S.C. 122 and 37 CFR 1.14. This collection is estimated to take 1.0 hour to complete, including gathering, preparing, and submitting the completed application form to the USPTO. Time will vary depending upon the individual case. Any comments on the amount of time you require to complete this form and/or suggestions for reducing this burden, should be sent to the Chief Information Officer, U.S. Patent and Trademark Office, U.S. Department of Commerce, P.O. Box 1450, Alexandria, VA 22313-1450. DO NOT SEND FEES OR COMPLETED FORMS TO THIS ADDRESS. SEND TO: **Attention Certificate of Corrections Branch, Commissioner for Patents, P.O. Box 1450, Alexandria, VA 22313-1450.**

*If you need assistance in completing the form, call 1-800-PTO-9199 and select option 2.*

APR 2 - 2007





## UNITED STATES PATENT AND TRADEMARK OFFICE

UNITED STATES DEPARTMENT OF COMMERCE  
 United States Patent and Trademark Office  
 Address: COMMISSIONER FOR PATENTS  
 P.O. Box 1450  
 Alexandria, Virginia 22313-1450  
 www.uspto.gov

APPL NO.	FILING OR 371 (c) DATE	ART UNIT	FIL FEE REC'D	ATTY. DOCKET NO	DRAWINGS	TOT CLMS	IND CLMS
09/965,236	09/26/2001	2623	664	205,286	6	21	2

ABELMAN FRAYNE & SCHWAB  
 Attorneys at Law  
 150 East 42nd Street  
 New York, NY 10017

CONFIRMATION NO. 7889

CORRECTED FILING RECEIPT

\*OC000000018664894\*

\*OC000000018664894\*

Date Mailed: 05/02/2006

Receipt is acknowledged of this regular Patent Application. It will be considered in its order and you will be notified as to the results of the examination. Be sure to provide the U.S. APPLICATION NUMBER, FILING DATE, NAME OF APPLICANT, and TITLE OF INVENTION when inquiring about this application. Fees transmitted by check or draft are subject to collection. Please verify the accuracy of the data presented on this receipt. **If an error is noted on this Filing Receipt, please mail to the Commissioner for Patents P.O. Box 1450 Alexandria Va 22313-1450. Please provide a copy of this Filing Receipt with the changes noted thereon. If you received a "Notice to File Missing Parts" for this application, please submit any corrections to this Filing Receipt with your reply to the Notice. When the USPTO processes the reply to the Notice, the USPTO will generate another Filing Receipt incorporating the requested corrections (if appropriate).**

**Applicant(s)**

Carsten Steger, Eching, GERMANY;

**Power of Attorney:**

Thomas Spath--25928

**Domestic Priority data as claimed by applicant****Foreign Applications**

EUROPEAN PATENT OFFICE (EPO) 00120269.6 09/27/2000  
 EUROPEAN PATENT OFFICE (EPO) 00124295.7 11/14/2000

If Required, Foreign Filing License Granted: 11/05/2001

The country code and number of your priority application, to be used for filing abroad under the Paris Convention, is **US09/965,236**

Projected Publication Date: Not Applicable

Non-Publication Request: No

Early Publication Request: No

\*\* SMALL ENTITY \*\*

APR 2 - 2007

MAR 2 - 2007



## UNITED STATES PATENT AND TRADEMARK OFFICE

UNITED STATES DEPARTMENT OF COMMERCE  
United States Patent and Trademark Office  
Address: COMMISSIONER FOR PATENTS  
P.O. Box 1450  
Alexandria, Virginia 22313-1450  
www.uspto.gov



## NOTICE OF ALLOWANCE AND FEE(S) DUE

7590 05/09/2005

ABELMAN FRAYNE & SCHWAB  
Attorneys at Law  
150 East 42nd Street  
New York, NY 10017

EXAMINER

CHANG, JON CARLTON

ART UNIT

PAPER NUMBER

2623

DATE MAILED: 05/09/2005

APPLICATION NO.	FILING DATE	FIRST NAMED INVENTOR	ATTORNEY DOCKET NO.	CONFIRMATION NO.
09/965,236	09/26/2001	Carsten Steger	205,286	7889

TITLE OF INVENTION: SYSTEM AND METHOD FOR OBJECT RECOGNITION

APPLN. TYPE	SMALL ENTITY	ISSUE FEE	PUBLICATION FEE	TOTAL FEE(S) DUE	DATE DUE
nonprovisional	NO	\$1400	\$300	\$1700	08/09/2005

THE APPLICATION IDENTIFIED ABOVE HAS BEEN EXAMINED AND IS ALLOWED FOR ISSUANCE AS A PATENT. **PROSECUTION ON THE MERITS IS CLOSED.** THIS NOTICE OF ALLOWANCE IS NOT A GRANT OF PATENT RIGHTS. THIS APPLICATION IS SUBJECT TO WITHDRAWAL FROM ISSUE AT THE INITIATIVE OF THE OFFICE OR UPON PETITION BY THE APPLICANT. SEE 37 CFR 1.313 AND MPEP 1308.

THE ISSUE FEE AND PUBLICATION FEE (IF REQUIRED) MUST BE PAID WITHIN **THREE MONTHS** FROM THE MAILING DATE OF THIS NOTICE OR THIS APPLICATION SHALL BE REGARDED AS ABANDONED. **THIS STATUTORY PERIOD CANNOT BE EXTENDED.** SEE 35 U.S.C. 151. THE ISSUE FEE DUE INDICATED ABOVE REFLECTS A CREDIT FOR ANY PREVIOUSLY PAID ISSUE FEE APPLIED IN THIS APPLICATION. THE PTOL-85B (OR AN EQUIVALENT) MUST BE RETURNED WITHIN THIS PERIOD EVEN IF NO FEE IS DUE OR THE APPLICATION WILL BE REGARDED AS ABANDONED.

## HOW TO REPLY TO THIS NOTICE:

I. Review the SMALL ENTITY status shown above.

If the SMALL ENTITY is shown as YES, verify your current SMALL ENTITY status:

- A. If the status is the same, pay the TOTAL FEE(S) DUE shown above.
- B. If the status above is to be removed, check box 5b on Part B - Fee(s) Transmittal and pay the PUBLICATION FEE (if required) and twice the amount of the ISSUE FEE shown above, or

If the SMALL ENTITY is shown as NO:

- A. Pay TOTAL FEE(S) DUE shown above, or
- B. If applicant claimed SMALL ENTITY status before, or is now claiming SMALL ENTITY status, check box 5a on Part B - Fee(s) Transmittal and pay the PUBLICATION FEE (if required) and 1/2 the ISSUE FEE shown above.

II. PART B - FEE(S) TRANSMITTAL should be completed and returned to the United States Patent and Trademark Office (USPTO) with your ISSUE FEE and PUBLICATION FEE (if required). Even if the fee(s) have already been paid, Part B - Fee(s) Transmittal should be completed and returned. If you are charging the fee(s) to your deposit account, section "4b" of Part B - Fee(s) Transmittal should be completed and an extra copy of the form should be submitted.

III. All communications regarding this application must give the application number. Please direct all communications prior to issuance to Mail Stop ISSUE FEE unless advised to the contrary.

**IMPORTANT REMINDER:** Utility patents issuing on applications filed on or after Dec. 12, 1980 may require payment of maintenance fees. It is patentee's responsibility to ensure timely payment of maintenance fees when due.

APR 21 - 2007

in the image. If the interior orientation of the camera is unknown, a perspective projection between two planes (i.e., the surface of the object and the image plane) can be described by a  $3 \times 3$  matrix in homogeneous coordinates:

$$\begin{pmatrix} x' \\ y' \\ t' \end{pmatrix} = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ t \end{pmatrix}$$

- 5 The matrix and vectors are only determined up to an overall scale factor (see Hartley and Zisserman (2000) [Richard Hartley and Andrew Zisserman: Multiple View Geometry in Computer Vision. Cambridge University Press, 2000], chapters 1.1–1.4). Hence, the matrix, which determines the pose of the object, has eight degrees of freedom. If the interior orientation of the camera is known, these eight degrees of freedom reduce to the six degrees of freedom of the pose of the object with respect to the camera (three for translation and three for rotation).

Often, this type of transformation is approximated by a general 2D affine transformation, i.e., a transformation where the output points  $(x', y')^T$  are obtained from the input points  $(x, y)^T$  by the following formula:

15 
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}.$$

General affine transformations can, for example, be decomposed into the following, geometrically intuitive, transformations: A scaling of the original  $x$  and  $y$  axes by different scaling factors  $s_x$  and  $s_y$ , a skew transformation of the  $y$  axis with respect to the  $x$  axis, i.e., a rotation of the  $y$  axis by an angle  $\theta$ , while the  $x$  axis is kept fixed, a rotation of both axes by an angle  $\phi$  and finally a translation by a vector  $(t_x, t_y)^T$ .

20 Therefore, an arbitrary affine transformation can be written as:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} 1 & -\sin\theta \\ 0 & \cos\theta \end{pmatrix} \begin{pmatrix} s_x & 0 \\ 0 & s_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}.$$

Figure 1 displays the parameters of a general affine transformation graphically. Here, a square of side length 1 is transformed into a parallelogram. Similarity transformations are a special case of affine transformations in which the skew angle  $\theta$  is 0 and both scaling factors are identical, i.e.,  $s_x = s_y = s$ . Likewise, rigid transformations are a special case of similarity transformations in which the scaling factor is 1, i.e.,  $s = 1$ . Finally, translations are a special case of rigid transformations in which  $\phi = 0$ . The relevant parameters of the class of geometrical transformations will be referred to as the pose of the object in the image. For example, for rigid

transformations the pose consists of the rotation angle  $\phi$  and the translation vector  $(t_x, t_y)^T$ . Object recognition hence is the determination of the poses of all instances of the model in the image.

Several methods have been proposed in the art to recognize objects in images. Most of them suffer from the restriction that the model will not be found in the image if it is occluded or degraded by additional clutter objects. Furthermore, most of the existing methods will not detect the model if the image exhibits non-linear contrast changes, e.g., due to illumination changes.

All of the known object recognition methods generate an internal representation of the model in memory at the time the model is generated. To recognize the model in the image, in most methods the model is systematically compared to the image using all allowable degrees of freedom of the chosen class of transformations for the pose of the object (see, e.g., Borgefors (1988) [Gunilla Borgefors. Hierarchical chamfer matching: A parametric edge matching algorithm. *IEEE Transactions on*

$$m = \frac{1}{n} \sum_{i=1}^n \langle d'_i, e_{x+p_i^x, y+p_i^y} \rangle = \frac{1}{n} \sum_{i=1}^n d_i'^x e_{x+p_i^x, y+p_i^y}^x + d_i'^y e_{x+p_i^x, y+p_i^y}^y.$$

- The advantage of this match metric is that neither the model image nor the image in which the model should be recognized need to be segmented (binarized), i.e., it suffices to use a filtering operation that only returns direction vectors instead of an extraction operation which also segments the image. Therefore, if the model is generated by edge or line filtering, and the image is preprocessed in the same manner, this match metric fulfills the requirements of robustness to occlusion and clutter. If parts of the object are missing in the image, there are no lines or edges at the corresponding positions of the model in the image, i.e., the direction vectors  $e_{x+p_i^x, y+p_i^y}$  will have a small length and hence contribute little to the sum. Likewise, if there are clutter lines or edges in the image, there will either be no point in the model at the clutter position or it will have a small length, which means it will contribute little to the sum. Therefore, the above match metric reflects how well the points in the image and model that correspond to each other align geometrically.
- However, with the above match metric, if the image brightness is changed, e.g., by a constant factor, the match metric changes by the same amount. Therefore, it is preferred to modify the match metric. By calculating the sum of the normalized dot product of the direction vectors of the transformed model and the image over all points of the model, i.e.:

$$m = \frac{1}{n} \sum_{i=1}^n \frac{\langle d'_i, e_{x+p_i^x, y+p_i^y} \rangle}{\|d'_i\| \cdot \|e_{x+p_i^x, y+p_i^y}\|} = \frac{1}{n} \sum_{i=1}^n \frac{d_i'^x e_{x+p_i^x, y+p_i^y}^x + d_i'^y e_{x+p_i^x, y+p_i^y}^y}{\sqrt{(d_i'^x)^2 + (d_i'^y)^2} \cdot \sqrt{(e_{x+p_i^x, y+p_i^y}^x)^2 + (e_{x+p_i^x, y+p_i^y}^y)^2}}.$$

Because of the normalization of the direction vectors, this match metric is additionally invariant to arbitrary illumination changes. In this preferred embodiment

All three normalized match metrics have the property that they return a number smaller than 1 as the score of a potential match. In all cases, a score of 1 indicates a perfect match between the model and the image. Furthermore, the score roughly corresponds to the portion of the model that is visible in the image. For example if the object is 50% occluded, the score cannot exceed 0.5. This is a highly desirable property because it gives the user the means to select an intuitive threshold for when an object should be considered as recognized.

Since the dot product of the direction vectors is related to the angle the direction vectors enclose by the arc cosine function, other match metrics could be defined that also capture the geometrical meaning of the above match metrics. One such metric is to sum up the absolute values of the angles that the direction vectors in the model and the direction vectors in the image enclose. In this case, the match metric would return values greater or equal to zero, with a value of zero indicating a perfect match. In this case, the pose of the model must be determined from the minimum of the match metric.

### Object Recognition Method

To find the object in the image, the a-priori unbounded search space needs to be bounded. This is achieved through the user by setting thresholds for the parameters of the search space. Therefore, in case of affine transformations the user specifies thresholds for the two scaling factors, the skew angle, and the rotation angle:

$$s_{x,\min} \leq s_x \leq s_{x,\max}$$

$$s_{y,\min} \leq s_y \leq s_{y,\max}$$

$$\theta_{\min} \leq \theta \leq \theta_{\max}$$

$$\varphi_{\min} \leq \varphi \leq \varphi_{\max}$$

e.g., the Gaussian filter used in the Canny edge extractor and the Steger line detector.

The transformation space needs to be discretized in a manner that the above requirement of all model points lying at most  $k$  pixels from the instance in the image can be ensured. Figure 3 shows a sample model of a key along with the parameters that are used to derive the discretization step lengths. The point  $c$  is the reference point of the model, e.g., its center of gravity. The distance  $d_{\max}$  is the largest distance of all model points from the reference point. The distance  $d_x$  is the largest distance of all model points from the reference point measured in the  $x$  direction only, i.e., only the  $x$  coordinates of the model points are used to measure the distances. Likewise, the distance  $d_y$  is largest distance of all model points from the reference point measured in the  $y$  direction only. To ensure that all model points created by scaling the model in the  $x$  direction lie within  $k$  pixels from the instance in the image, the step length  $\Delta s_x$  must be chosen as  $\Delta s_x = k/d_x$ . Likewise,  $\Delta s_y$  must be chosen as  $\Delta s_y = k/d_y$ . The discretization of the skew angle only depends on the distance  $d_y$ , since the  $x$  axis remains fixed in a skew operation. Hence the step length of the skew angle  $\Delta\theta$  must be chosen as  $\Delta\theta = \arccos(1 - k^2/(2d_y^2))$ . Similarly, the step length of the rotation angle must be chosen as  $\Delta\phi = \arccos(1 - k^2/(2d_{\max}^2))$ . Finally, the step lengths in the translation parameters must both be equal to  $k$ , i.e.,

$\Delta t_x = \Delta t_y = k$ .

$l$	$\Delta t_x$	$\Delta t_y$	$\Delta\phi$
0	1	1	0.573
1	2	2	1.146
2	4	4	2.292
3	8	8	4.585

For a further speed-up, the number of points in the model is preferably also reduced by a factor  $k = 2^l$  for the different discretization levels of the search space (see the section on model generation below). The necessary smoothing is preferably

5 obtained by using a mean filter, because it can be implemented recursively, and hence the runtime to smooth the image does not depend on the smoothing parameter. Alternatively, a recursive implementation of the Gaussian smoothing filter can be used. Although this discretization and smoothing method already yields acceptable runtimes, a further speed up can be obtained by also subsampling the

10 image by a factor identical to the translation step lengths  $\Delta t_x$  and  $\Delta t_y$  on each discretization level. In this case, the step lengths for the translation parameters will, of course, have to be set to 1 in each level. Now, however, care must be taken to propagate the correct translation parameters through the levels of discretization. If, as described above, the translation step lengths double in each level of the

15 discretization space, the translation parameters of the found objects must be



below in the section on model generation, the transformed models may be precomputed at the time the model is generated. If this was not done, the model must be transformed in this step by applying the affine transformation parameters to the points of the model and the linear transformation parameters to the direction  
5 vectors of the model. This results in a score for each possible combination of parameter values. The scores are then compared with the user-selected threshold  $m_{\min}$ . All scores exceeding this threshold are combined into regions in the search space. In these regions, local maxima of the match metric are computed by comparing the scores of a certain set of parameters with scores of neighboring  
10 transformation parameters. The resulting local maxima correspond to the found instances of the model in the coarsest discretization of the search space. These found instances are inserted into a list of instances, which is sorted by the score of the instance.

Once the exhaustive match on the coarsest discretization level is complete, the  
15 found instances are tracked through the finer levels of the discretization space until they are found at the lowest level of the discretization space (step 5). The tracking is performed as follows: The first unprocessed model instance is removed from the list of model instances. This is the unprocessed instance with the best score, since the list of instances is sorted by the score. The pose parameters of this instance are then  
20 used to define a search space in the next lower level of the discretization. Ideally, the model would be located at the position given by the appropriate transformation of the pose parameters, i.e., the scaling parameters  $s_x$  and  $s_y$ , as well as the angles  $\phi$  and  $\theta$  are rounded to the closest parameter in the next finer level of the discretization, while the translation parameters are either scaled by a factor of 2, if image pyramids

are used, or passed unmodified, if subsampling is not used. However, since the instance has been found in a coarse discretization level in which the image has been smoothed by twice the amount than in the next finer level, there is an uncertainty in the pose parameters that must be taken into account when forming the search space in the next lower level of the discretization. A good choice for the search space is obtained by constructing a rectangle of size  $5 \times 5$  around the propagated translation parameters. Furthermore, the search space for the other four parameters is constructed by including the next lower and higher values of the parameters in the finer level into the search space. As an example, suppose the space of transformations consists of the rigid transformations, that image pyramids have been used, and the instance has been found in level  $l=3$  of the discretization with the following pose:  $(t_x, t_y) = (34, 27)$   $\phi = 55.020^\circ$ . Then the search space in the finer level  $l=2$  is given by:  $66 \leq t_x \leq 70$ ,  $52 \leq t_y \leq 56$ , and  $52.716^\circ \leq \phi \leq 57.300^\circ$  (the table with the discretization step lengths in the example above should be kept in mind). The model is then searched with all transformations in the search space in the finer level by computing the match metric in the same manner as described above for the exhaustive match on the coarsest level of discretization. The maximum score within the search space is identified. If the corresponding pose lies at the border of the search space, the search space is iteratively enlarged at that border until the pose with the maximum score lies completely within the search space, i.e., not at the borders of the search space. If the maximum score thus obtained exceeds the user-selected threshold  $m_{\min}$ , the instance is added to the list of found instances in the appropriate place according to its score.

$$\frac{1}{m} \sum_{j=1}^m \|Aq_j + t - r_j\|_2 \rightarrow \min$$

With this approach, the model and image points must be extracted with subpixel precision. If they are only extracted with pixel precision, the model points on average cannot be moved closer to the image points than approximately 0.25 pixels because of the discrete nature of the model and image points, and hence no improvement in the accuracy of the pose would result. However, even if the model and image points are extracted with subpixel precision the model and the image cannot be registered perfectly because usually the image and model points will be offset laterally, which typically results in a nonzero average distance even if the model and the found instance would align perfectly. Furthermore, the traditional least-squares approach neglects the direction information inherent in the model and the image. These shortcomings can be overcome by minimizing the distance of the image points from the line through the corresponding model point that is perpendicular to the direction stored in the model. For edges and lines, this line is parallel to the model edge or line. The line through the model point in the direction perpendicular to the model direction vector is given by  $d_j^T(p - p_j) = d_j^x(x - p_j^x) + d_j^y(y - p_j^y) = 0$ . Therefore, the following distance would need to be minimized:

$$\frac{1}{m} \sum_{j=1}^m \left( A d_j^T (r_j - (A q_j + t)) \right)^2 = \frac{1}{m} \sum_{j=1}^m \left( d_j^{x'} (r_j^x - q_j^{x'} - t_x) + d_j^{y'} (r_j^y - q_j^{y'} - t_y) \right)^2 \rightarrow \min$$

An approach of this type for determining a rigid transformation is described in Wallack and Manocha (1998) [Aaron Wallack and Dinesh Manocha. Robust Algorithms for Object Localization. *International Journal of Computer Vision*, 27(3):243–262, 1998]. This approach assumes that the correspondence problem has already been solved. Furthermore, the model features are line segments and circular arcs instead of points and direction vectors. However, this approach is computationally inefficient because both the model points and their direction vectors need to be transformed. Approximately half of the operations can be saved if instead the transformation from the image points to the model points is computed in the least-squares fit, i.e.,

of the search space. If an image pyramid has been used in step (2) to transform the image, this reduction of the number of points in the model happens automatically. If the subsampling was not performed, the number of data points is reduced after the feature extraction in step (6) below.

- 5 For each level of discretization, the search space is sampled according to the discussion of the object recognition method above, using user-specified bounds on the linear transformation parameters:

$$s_{x,\min} \leq s_x \leq s_{x,\max}$$

$$s_{y,\min} \leq s_y \leq s_{y,\max}$$

$$\theta_{\min} \leq \theta \leq \theta_{\max}$$

$$\phi_{\min} \leq \phi \leq \phi_{\max}$$

The translation parameters are not sampled, i.e., fixed translation parameters.

- 10  $t_x = t_y = 0$  are used, because the translation parameters do not change the shape of the model. The steps (5)–(7) are performed for each set of parameters from the sampled search space for the current level of discretization. The reason for sampling the search space is to precompute all possible shapes of the model under the allowable transformations and to store the in memory, leading to a significant
- 15 reduction of runtime in the object recognition phase.

In step (5), the transformed image of the current level of discretization, i.e., the image at the current level of the pyramid or the appropriately smoothed image, which was generated in step (2), is transformed with the current transformation parameters.

- Here, care must be taken that the object still lies completely within the image after the image transformation. If necessary, a translation is added to the transformation
- 20 to achieve this, which is accounted for when the extracted model points are added to

the object can be subsampled to generate a model with fewer model points, i.e., only every k-th point of the contour is added to the model.

Finally, the model points obtained in step (6) are added to the collection of models at the current discretization level, along with the transformation parameters that were used to generate the transformed model. To make the matching more robust if a greedy search strategy is used, it is useful to add the points to the model in an order in which the first few points of the model are distributed well across the model. This is necessary in the greedy strategy because if all points in the first part of the model happen to be occluded, while all other points would be present, the matching strategy may not find the instance of the model. The simplest way to achieve an even distribution of the first model points is to add the model points to the model in a randomized order.

The model generation strategy described above may generate a very large number of precomputed models of the search space of allowable transformations is large. This leads to the fact that the memory required to store the precomputed models will be very large, which either means that the model cannot be stored in memory or must be paged to disk on systems that support virtual memory. In the second case, the object recognition will be slowed down because the parts of the model that are needed in the recognition phase must be paged back into the main memory from disk. Therefore, if the memory required to store the precomputed models becomes too large, an alternative model generation strategy is to omit step (4) of the method above, and instead to compute only one precomputed model of the object at each level of discretization, corresponding to transformation parameters that leave the object unchanged, i.e.,  $s_x = s_y = 1$  and  $\phi = \theta = 0^\circ$ . In this case, the transformation of